**Bonds – Chapt. 8 in RWJJ**

**Definitions:**

**Bond** - An IOU. A loan. - A security that obligates the issuer to make specified payments to the bondholder.

**Maturity** - Date when the loan is paid off.

**Face Value** – Large single-sum payment at maturity – separate from an interest payment

**Coupon Rate** - Annual interest payment as a percent of the face value

Note that most bonds make their interest payments semiannually.

**Example**: 10 year $10,000 U.S. Treasury Note with 7% coupon rate. New Issue

Maturity – Ten years from today

Face Value - $10,000

Coupon Rate - 7% of $10,000 = $700 per year = $350 every six months

350 350 350 350 350 350 350 350 350 350 350 350 350 350 350 350 350 350 350 10,350

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

**Who issues Bonds?**

US Government - bills(< 1yr.), notes(1-10 yrs) and bonds(> 10 yrs)

Other governments

Corporations

States

Cities

School Districts

Hospitals

**What is a bond worth?**

Price of Bond = Value of Bond

The value of the bond is the present value of its future cash flows, discounted at the appropriate (appropriate based on risk) current interest rate (the bond’s yield)

**Example**: The above bond is a 10 year annuity (20 semiannual coupon payments) plus the $10,000 face value. Assume the current interest rate (bond’s yield) is 7%. What is its price?

If we have semiannual payments, we need to use the semiannual interest rate and the number of semiannual payments, so r = 7%/2 = 3.5% and t = 20



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= 10,000

If you hold the bond for 2 years and the interest rate (yield) is still 7%, what is the bond’s value? Note there are now 16 semiannual periods remaining.

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= 10,000

Each year the PV of the interest payments decreases (due to fewer payments) but the PV of the principal increases (because you are closer to the time you get it). As long as the bond’s coupon rate is equal to the bond’s yield (the current interest rate), the bond’s price will remain equal to its face value.

Now suppose we hold the bond for 2 years and comparable investments elsewhere are now paying 5% (2.5% semiannually). Note the bond itself has not changed.



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= $11,305.50 Note that the price went up

Why did the price of the bond go up?

Look at the same thing, but comparable investments are now 9% (4.5% semiannually).



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= $8,876.60 Note that the price went down

Why did the price of the bond go down?

# Bond values and interest rates move in opposite directions

Bond prices are usually expressed as a percentage of their face value.

**Examples**: 100 - Selling at par

102 - Selling at a premium (102% of the face value)

98 - Selling at a discount (98% of the face value)

**Premium** – Price is above the face value

**Discount** – Price is below the face value

All else equal, bonds with a higher coupon rate will sell at a premium and those with a lower coupon rate will sell at a discount.

**Yield to Maturity** - Also the IRR or BEY (Bond Equivalent Yield) of the bond.

The interest rate for which the PV of the bond’s future cash flows equals its price.

What interest rate solves this?



Note the same r appears four times in the equation.

The solution is 2.5%. With semiannual payments, this is the semiannual yield. By convention, we double this to get the yield to maturity of 5%. By definition, the YTM of a bond is its semi-annual yield times two. The effective annual rate (EAR) would be greater than the YTM. The YTM is the same idea as an APR.

You can solve this with a financial calculator, Excel, or by trial and error.

Solve for r using Excel’s Rate Funtion:

PV = -11,305.50

PMT = 350

FV = 10,000

t = 16

The solution is 2.5% which we double to get a YTM of 5.00%.

On Excel, if you know the exact dates (settlement and maturity dates), you can use the Yield Function.

**Yield to maturity** is the rate of return you get if you hold the bond to maturity (and if you are able to reinvest all coupon payments at that rate).

Similar bonds (risk and maturity) should have similar YTMs. In a competitive marketplace, prices adjust to give similar bonds the same YTM. - If one bond has a higher yield, people buy it which bids up the price, and its yield falls.

**More Yield to Maturity:**

$10,000 10-year bond with 2 years remaining till maturity

Coupon Rate = 7%

Price = $10,370.00

What is the yield to maturity?

$10,370.00 = 350 + 350 + 350 + 350 + 10,000

(1+r) (1+r)2 (1+r)3 (1+r)4 (1+r)4

Note that if the price is > $10,000, the interest rates must have gone down from 7%.

In this case, r = 2.52%, and the yield is 5.03%

**Interest Rate Risk** - The risk of fluctuations in a bond’s value due to interest rate changes.

Note: If interest rates go up, the value of every bond goes down, but some bonds lose more value than other bonds. The biggest factor affecting the interest rate risk of a bond is its time to maturity. Which bond has more interest rate risk: a 2-year bond, or a 20-year bond?

Can the price of a bond change even if interest rates don’t?

**Example**: You bought this 7% 10-year bond 2 years after it was issued for $11,305.50. Its yield to maturity was 5%. What is it worth in 3 years if it still yields 5%?

\_\_\_\_\_\_\_\_\_\_\_\_\_11,305.50\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_?\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_maturity

0 4 10 20



Price = 10,875.20

Why did the price drop?

The prices of all bonds will converge to their face amount as they approach maturity. Of course if the price *was* the face amount (par value), it remains constant.

Actually, every bond has two prices:

**Ask** - Price a bond dealer is willing to sell a bond for

**Bid** - Price a bond dealer is willing to buy a bond for

Ask > Bid

**Ask – Bid**: The difference is the **spread** - The profit to the dealer.

When we don’t specify whether we are referring to the bid or the ask, we can assume that “the price” of a bond is the midpoint between the bid and ask prices.

**Government Bonds** - Prices are often quoted in 32nds.

Example: 105:28 or 105-28 = 105 28/32 % of face value

= 105.875% of face value

For $1 million bond = $1,058,750

**Term Structure of Interest Rates** - The relationship between time to maturity and yield to maturity. Do long-term bonds yield more, less, or the same as short-term bonds? - That’s the question the term structure addresses.

**Yield Curve** - A graph of the term structure with YTM on vertical axis and time to maturity on the horizontal axis.

Yield Curves can be:

Upward Sloping

Downward Sloping

Flat

An upward sloping is the most typically seen of the three.

**Default Risk (or credit risk)** - The risk that whoever we loaned money to will not pay it back on time or in full.

**Bond Ratings** - Provided by Moody’s and Standard & Poor’s (S&P). The ratings are not identical, but they are close. Companies and municipalities pay to be rated.

Ceteris Paribus, the highest grade bond offers the lowest return.

There is a risk/return tradeoff.

**Inflation**

Inflation means a drop in the purchasing power of money. If goods cost $100 last year and $103 today, we have 3% inflation.

**CPI** - Consumer Price Index - The most commonly used measure of inflation. It is based on changes in prices of a standard basket of goods. Some say that it overstates the inflation rate because it doesn’t allow for substitution.

You calculate inflation the same way you calculate interest.

3% inflation for 10 years is (1.03)10

$100 of goods in 10 years will cost $134.39

Inflation must be considered in investment decisions.

**Nominal cash flow** - The actual number of dollars to be received in the future (adjusted upwards to reflect expected inflation).

**Real cash flow** - Expressed in terms of time zero purchasing power.

**Nominal Interest Rate** - The rate of growth of your money

**Real Interest Rate** - The rate of growth of your purchasing power

**Fisher Equation:**

1 + Real Interest Rate = 1 + Nominal Interest Rate

1 + Inflation Rate

**Approximation**: Real Interest Rate = Nominal Interest Rate – Inflation rate

**Example**:

Nominal interest rate = 7%

Inflation = 3%

What is the real interest rate?

1.07 = 1.0388 ⇒ Real rate = 3.88%

1.03

Approximation: 7% - 3% = 4%

If inflation expectations increase, nominal interest rates increase in order to maintain purchasing power. Note: we often refer to inflation, but it’s really inflation expectations since the interest rates declared in the present are matched with inflation expectations for the future.

To convert nominal dollars to real dollars:

Real Dollars = Nominal Dollars / (1+inflation rate)t

To convert real dollars to nominal dollars:

Nominal Dollars = Real Dollars (1+inflation rate)t

You must account for inflation in any problem involving the time value of money.

You must either adjust the dollars or the interest rate.

Nominal cash flows must always be discounted at the nominal rate.

Real cash flows must always be discounted at the real rate.

Note that at time zero, nominal dollars equals real dollars.

An important time to consider inflation is in **retirement planning**.

Suppose I’m 25 and plan to retire at age 65

I want $60,000/yr for 35 years (I think I’ll live to age 100), but what I really want is what $60,000/yr can buy me today (the purchasing power of $60,000 each year)

Assumptions:

* Inflation is expected to be level at 2%
* Interest (investment) rates are 7%

How much must I save per year?

Real Interest Rate: 1.07 = 1.049 ⇒ real return = 4.9%

1.02

To get a $60,000/yr annuity for 35 years – in real dollars:

PVA = 

= $994,725.79

So I need $994,725.79 of today’s dollars at age 65 to be able to consume $60,000 (today’s dollars) per year for 35 years.

To get $994,725.79 of today’s purchasing power, what I’ll actually need is $2,196,393.99 at age 65. This means converting Real Dollars to Nominal Dollars.

994,725.79 (1.02)40 = 2,196,393.99

I must save $\_\_\_/yr. to get to $2,196,393.99 in 40 years.

We can most easily do this as the Future value of an annuity.

2,196,393.99 = C [(1.07)40 – 1]

.07

C = $11,002.04

So, if I save $11,002.04 (nominal dollars) per year for 40 yrs. at 7%, I’ll have $2,196,393.99.

But due to inflation, the $2,196,393.99 will only be worth the equivalent of $994,725.79.

However, this will be enough to generate (at 7% interest with 2% inflation), a 35-year annuity worth the equivalent of $60,000 per year.

How much money will I withdraw from my retirement account when I turn 66?

$60,000 in real dollars

$60,000 (1.02)41 = $135,132.03 in nominal dollars

You will then withdraw 2% more (in nominal dollars) each year to keep up with inflation.

I can also determine how many real dollars I need to invest into my retirement account each year. This is a bit more complex because it means I have to invest a different amount of money (in nominal dollars) each year, but it means that I’m setting aside the same purchasing power each year.

Discount real dollars at the real interest rate:

994,725.79 = C [(1.049)40 – 1]

.049

C = 8,433.56

So each year, you set aside $8,433.56 of today’s purchasing power.

In nominal dollars, this is:

$8,433.56 (1.02) = $8,602.23 at age 26

$8,433.56 (1.02)2 = $8,774.28 at age 27

$8,433.56 (1.02)3 = $8,949.76 at age 28

etc.