**Explanations of Quiz Solutions for Chapters 10-11**

1. Since these five years are only as sample of the entire population of returns, we can never know the true means, variances, standard deviations, or covariances. We must calculate estimates using the sample population that is given to us (if we believe that it is a randomly sample that is representative of the entire population of returns). To find the sample mean returns, just add the five yearly returns together and divide by the number of years (5) to get 10% for X and 16.2% for Y. This is our estimate of the mean (average) return for the entire population of returns for each company. Since variance measures how far the individual returns tend to vary from the average, we need to measure how far each data point (yearly return) is from the sample mean that we just calculated. We square that distance, sum them up and divide by N-1 (5-1=4 in this example). If we had ALL the data points in the entire population, we would divide by N to find the variance of the population. But since we don’t have all the data points, we will never know the true variance and we are only calculating an estimate of the variance. For X, this is .01685 and for Y it is .06167. The estimate of the standard deviation (standard deviation of the sample) is the square root of the sample variance. To find the sample covariance between X and Y, we essentially combine the calculations of each stock’s variance. Instead of squaring the difference from a data point (in X) to its sample mean, we multiply it by the difference from the data point in Y to its sample mean for the same year (hence the name co-variance). Again, we divide by N-1 because we do not have all the data to know the true covariance. We see that X and Y tend to move in opposite directions of their respective sample means. They have a sample covariance of -.010025. We can standardize this value by dividing it by the product of the sample standard deviations of X and Y. This gives us a value on the scale of -1.0 to +1.0. The Correlation between X and Y is -.311.

Excel has built-in functions for each of these statistics. To solve for the sample arithmetic mean, use AVERAGE. To solve for the sample variance, use VAR.S. To solve for the sample standard deviation, use STDEV.S. To solve for the sample covariance, use COVARIANCE.S. To solve for the correlation coefficient, use CORREL. If you have an earlier version of Excel than 2010, the names of these functions may be different.

1. Since we want the arithmetic mean for this six-year period and we have all six years, we can find the true mean – we have all the data. We are not trying to estimate the mean for a larger time period. We find the arithmetic mean simply by adding the data points together and dividing by the number of data points (6) to get 8.83%. The geometric mean (buy-and-hold-return) is found by adding one to each of the annual returns and multiplying these values together. That product is raised to the 1/N value (1/6 in this case), and one is subtracted from the result. This gives us a geometric mean of 7.69%. In Excel, use AVERAGE to find the arithmetic mean, but do not use Excel’s built-in function for geometric mean as it gives incorrect answers when negative values are included. You can, however, add one to each value, use the GEOMEAN function and then subtract one to get the correct answer. Since we have all the data and it happened in the past (these are not future probabilities), we can find the standard deviation in the usual manner by adding up the squared value of the distance each data-point is from the arithmetic mean and dividing the sum by the sample size (six in this case). Excel will also give us the standard deviation, but be sure you are calculating the standard deviation of a population, not a sample.
2. A total return is not time-dependent. It is simply found by subtracting the starting amount from the end amount and dividing by the starting amount (or dividing the end amount by the starting amount and then subtracting one). In this example, the S&P 500 was up 3.55% over the time period.

When we assume semiannual compounding, we assume that the S&P 500 was at 1,352 from January 1 through June 30. Then, on July 1, it jumped up to 1,400. So if this were to happen in every semiannual period contained in a year (there are two of them), the annual return would be 7.23%. This is found exactly as we found the Effective Annual Rate (EAR) in chapter 4. Add one to the semiannual return, raise it to the 2nd power (since there are two semiannual periods in a year), and then subtract one.

If we assume continuous compounding, we assume that the S&P 500 was always (continuously) increasing between January 1 and June 30. Every moment of every day, it was steadily increasing at the same compounded rate. Since this is a semiannual period (January 1 – July 1), we first find the semiannual compounded rate by dividing the end value by the beginning value and taking the natural log of the result. Dividing 1400 by 1352 gives us 1.035503. The natural log (use the LN function in Excel) of 1.035503 is 0.034887 (3.4887%), meaning that e raised to the 0.034887 power gives us 1.035503. With continuous compounding, if you want to annualize something that took place over less than a year, simply multiply the result by how many time periods of it there are in a year. In this example, multiplying the 3.4887% continuously compounded semiannual return by 2 gives us an annualized return of 6.98%.

1. The expected return for the market (or any asset for that matter) is the risk-free rate of return plus a risk premium for that asset. In this problem, both the risk-free rate and the risk premium for the market are given to us, so the expected return for the market is 9.2%. This, of course, does not mean that we are certain that the market will return 9.2% this year. It means that the actual return is a random variable with a normal distribution where the mean (expected) value is 9.2% and the standard deviation (the square root of the variance) is 22%. If we know the mean and standard deviation of any normal distribution, Excel can find the area to the left of any value under the curve through its NORMDIST function. Since losing money means earning a return that is less than zero, we enter 0 for X in the function, 9.2% for MEAN, and 22% for STANDARD\_dev. For CUMULATIVE, enter “True”. Excel tells us that with 9.2% (.092) at the midpoint of the curve; the proportion of the curve to the left of zero is .338 (33.8%), so there is a 33.8% probability that the market will return less than a 0% return next year.
2. The weight in each asset is the proportion of your money that is invested in it. Since you are buying 100 shares of Coca-Cola (KO) at $60 each, you are investing $6,000 into KO. General Electric (GE) and Microsoft (MSFT) are each priced at $30, so 100 shares of each will cost you $3,000 for each stock. Your total investment is $12,000 which means the weight on KO is .5 with the weights on GE and MSFT being .25 each.

The expected return of a portfolio is always a weighted average of the expected returns of the individual assets with each asset’s expected return weighted by the proportion of money you have invested in that asset. So we multiply the .5 weight for KO by its expected return of 16% (.16), add to it the .25 weight of GE times its expected return of 12%, and add to it the .25 weight of MSFT times its expected return of 14%. The resulting expected return of the portfolio is 14.5%.

The standard deviation of a portfolio is the square root of the portfolio’s variance. The variance of a portfolio is the sum of the cells in its weighted variance/covariance matrix. With 3 assets in the portfolio, you will have (3x3) 9 cells in the matrix. Of these 9 cells, 3 will be variance cells (one for each asset) and the remaining 6 will be covariance cells. If you draw out a 3x3 matrix, it is easy to see this. Since the covariance between KO and GE is the same thing as the covariance between GE and KO, we have 3 pairs of covariance terms: KO with GE, KO with MSFT, and GE with MSFT. In this problem, the covariances are not given, but can be easily calculated because the covariance between any two assets is equal to the correlation between those two assets times the product of their standard deviations. So the covariance between KO and GE is their correlation (0.5) times the standard deviation of KO (.05) times the standard deviation of GE (.08), which equals 0.002. When you multiply that covariance between KO and GE by the weight on KO (.5) times the weight on GE (.25), you get a value of 0.00025 which goes in that particular wtd var/cov cell. The variance terms are not given to us, but the standard deviations are, so since the standard deviation of KO is 5% (.05), its variance is (.05)2 = .0025. This gets multiplied by the weight on KO squared (.5)2 = .25 to give us a value of .000625 in that particular variance cell. When we add up all 9 cells, we get .00265375 which is the variance of the portfolio. Taking the standard deviation of the variance gives us a standard deviation of 5.15%.

1. The beta of a stock is its covariance with the market divided by the variance of the market. This is mathematically equivalent to saying that the beta is that stock’s correlation with the market multiplied by the ratio of the standard deviation of that stock divided by the standard deviation of the market. For Disney, we are given its correlation with the market, its standard deviation, and the standard deviation of the market, so we can easily calculate its beta. To find Disney’s expected return using the Capital Asset Pricing Model (CAPM), we need to use the current risk-free rate (not what it has been in the past), and the market risk-premium (RM - Rf), which is the extra return you expect the market to earn above the risk-free rate in the future. Disney’s returns over the past 10 years are irrelevant to this problem, as is the market’s year-to-date return and the risk-free rate in the past. The math on this problem is very easy. The key is to understand what data we are looking for to put into the CAPM.
2. The expected return of a portfolio is a weighted average of the expected returns of the stocks that make up the portfolio with each stock’s expected return being weighted by the proportion of money that you have invested into that stock. In this problem, you have 40% of your money invested into Stock X and 60% invested into Stock Y, so we weight the expected return of Stock X by .4 and the expected return of Stock Y by .6 to give us an expected return on the portfolio of 21%. Since the correlation between the stocks isn’t considered when calculating the expected return of the portfolio, 21% is the expected return when the correlation between X and Y is either 0.5 or -0.5.

The standard deviation of a portfolio is the square root of its variance. The variance of a portfolio is the sum of the cells in its weighted variance/covariance matrix. In this problem, there are two stocks in the portfolio, so we have a 2x2 matrix with 2 variance cells (one for each stock) and 2 covariance cells (both covariance cells are identical). The first variance cell has the weight on Stock X squared multiplied by the variance of Stock X. Since the standard deviation of Stock X is 40%, its variance is (.4)2 and since its weight in the portfolio is 40%, its weight squared is (.4)2. The second cell is the weight on Stock Y squared times its variance. Since the standard deviation of Stock Y is 65%, its variance is (.65)2 and since its weight in the portfolio is 60%, its weight squared is (.6)2. Since the two covariance cells are identical, we will describe the contents of one of them and multiply it by 2. A covariance cell will contain the weight of Stock X times the weight of Stock Y times the covariance between stocks X and Y. The covariance is not given to us directly, but we know that it equals the correlation between stocks X and Y multiplied by the standard deviation of Stock X and the standard deviation of Stock Y. The problem asks us to find the standard deviation of the portfolio first with a correlation of 0.5 and then with a correlation of -0.5. With a correlation of 0.5, the covariance between stocks X and Y is (.5)(.4)(.65) = .13. With a correlation of -.05, the covariance is (-.5)(.4)(.65) = -.13. In each case, the covariance is multiplied by the product of the weights of the two stocks (.4)(.6) to show what is in the covariance cell. As mentioned above, since there are two identical covariance cells, we multiply the components of a covariance cell by 2. This gives us a portfolio variance of .2401 when the correlation is .5 and a portfolio variance of .1153 when the correlation is -.5. In each case, we find the portfolio standard deviation by taking the square root of the portfolio variance. Note that the standard deviation of the portfolio is less when the correlation between the stocks is -0.5 than when it is 0.5.

1. The expected return of an asset is a weighted average of its possible returns in each state of the world weighted by the probability of that state occurring. For Stock J, there is a .25 probability that it will have a return of -2%, a .6 probability that it will have a return of 9.2%, and a .15 probability that it will have a return of 15.4%. Thus its expected return is 7.33%.

The standard deviation of an asset is the square root of its variance. When dealing with probabilities of future returns, the variance is a weighted average of the squared differences of the possible returns from the expected return. For Stock J, there is a .15 probability that it will have a return of 15.4%, so .15 is multiplied by the squared difference between 15.4% and stock A’s expected return of 7.33%. The same procedure is followed for a normal economy, where there is a .6 probability that stock J will have a return of 9.2%. The difference between 9.2% and the expected return of 7.33% is squared and then multiplied by the probability of .6. Notice that in any weighted average, the weights must always add up to one. Once we have the variance of stock J, we take the square root (raising something to the ½ power is the same thing as taking its square root) to get a standard deviation of 5.80%.

To find the covariance between stocks J and K we start by taking the probability of the first state of the economy (Bear – probability of .25) and multiplying it by the difference between J’s return in that state and J’s expected return (-.02 - .0733) and multiplying that by the difference between K’s return in that state and K’s expected return (.050 - .0608). We do the same thing for each state of the economy (Normal and Bull) and then add the terms together giving us a covariance of .000425.

The correlation between the two stocks is found by dividing their covariance by the product of their standard deviations. Since we found their covariance and both of their standard deviations earlier in this problem, this part can’t be done until after the earlier parts are successfully completed.

1. The expected return of a portfolio is a weighted average of the expected returns of the assets that comprise the portfolio. As with any weighted average, the weights must add up to one. This means that when there are only two assets in the portfolio, if we define the weight on the first asset as wA, the weight on the second asset can be defined as (1 – wA). So if we know the expected return on the portfolio (25%) and the expected returns for the individual assets (10% for A and 20% for B), we are left with only one unknown in our equation – the weight on asset A (wA). Using basic algebra we find that wA = -.5. A negative weighting on an asset means that we are shorting the asset (borrowing it and then selling it). When we sell an asset that we have borrowed, we can use the proceeds to purchase more of another asset than we would otherwise be able to. In this case, that means that we weight on Asset B will be 1.5. For example, if we have $100 to invest, we will put it all into Asset B. But we will also short Asset A by borrowing and then selling enough of it to raise an additional $50. That $50 is also invested into Asset B meaning that our total investment into Asset B is $150, or 150% of the money we have. Only through this use of leverage (shorting), can we obtain an expected return for a portfolio that is greater than the expected return of any of the individual assets which comprise the portfolio.
2. If you have the same amount of money invested into each stock in the portfolio (an equally-weighted portfolio), the weight on each stock must be 1/N where N is the number of stocks in the portfolio. For example, if you have 2 stocks in the portfolio, the weight on each is ½. If you have 3 stocks in the portfolio, the weight on each is 1/3, and so on. In this case, the weight on each stock is 1/20. The standard deviation of the portfolio is the square root of its variance. The variance of the portfolio is the sum of the cells in its weighted variance/covariance matrix. Since there are 20 stocks in the portfolio, the matrix will be 20x20 = 400 cells. Of the 400 cells, there will be 20 variance cells (one for each stock) and 380 covariance cells. Because each stock has the same standard deviation and the portfolio is equally weighted, each of the 20 variance cells will look the same. Each will contain the weight on the stock squared times the variance of the stock. With a weight of 1/20, the weight squared will be (1/20)2. With the standard deviation of each stock being 60%, the variance will be (.6)2. With 20 identical cells, the addition of them will be (20) (1/20)2 (.6)2. Since each pair of stocks has the same correlation (.1), and each stock has the same standard deviation, the covariance between each pair of stocks will be the same (.1) (.6) (.6) because the covariance between any two assets equals their correlation coefficient multiplied by each of their standard deviations. This means that each of the 380 covariance cells will look exactly the same. They will each have the weight on the first asset (1/20) times the weight on the second asset (1/20) times the covariance between the assets. Since there are 380 covariance cells, the sum of them will be (1/20) (1/20) (.1) (.6) (.6). So the sum of all 400 cells in the weighted variance/covariance matrix (and thus, the variance of the portfolio) will be the sum of the 20 variance cells added to the sum of the 380 covariance cells. This comes to .0522. The square root of the variance of the portfolio is the standard deviation of the portfolio which is 22.85%.